The persistence of waterfalls in fractured rock

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ABSTRACT

Although the upstream translation of waterfalls is often thought to occur by undercutting of resistant strata, collapse, and headwall retreat (e.g., Niagara Falls), many propagating waterfalls maintain a vertical face in the absence of undercutting. To explain this observation, we propose that bedrock-fracture geometry exerts a fundamental control on knickpoint morphology and evolution such that vertical waterfalls can persist during retreat due to toppling in bedrock with near horizontal and vertical sets of joints (e.g., columnar basalt). At a waterfall, rock columns are affected by shear and drag from the overlying water, buoyancy from the plunge pool at the foot of the waterfall, and gravity. We used a torque balance to determine the stability of a rock column and any individual blocks that comprise the column. Results indicate that rotational failure should occur about the base of a headwall (and therefore preserve its form during upstream propagation) where columns are tilted in the downstream direction or slightly tilted in the upstream direction, depending on the plunge-pool depth. Flume experiments were performed to test the model, and the model provides a good prediction of the flow necessary to induce toppling and the morphology of the headwall. Waterfall-induced toppling explains the morphology of canyon headwalls in the volcanic terrain of the northwestern United States, where catastrophic paleofloods (e.g., Bonneville Flood) have carved steep amphitheater-headed canyons in columnar basalt. This model may also explain similar landforms elsewhere on Earth and Mars, and it can be used to predict the minimum flow discharge needed to create these features.

Keywords: knickpoint, amphitheater, plucking, toppling, bedrock erosion, Mars.

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INTRODUCTION

The upstream propagation of steepened reaches in bedrock rivers (i.e., knickpoints) is one of the fundamental drivers of landscape evolution in hilly and mountainous terrain. Where knickpoints are near vertical, waterfalls form and in some cases can maintain their form while propagating upstream. Why do some waterfalls persist rather than decline in height or rotate as they propagate upstream? To answer this question, some researchers have adapted fluvial-erosion models based on bed stress, stream power, or saltating sediment to explore the development and translation of knickpoints (Howard et al., 1994; Seidl et al., 1994; Whipple and Tucker, 2002; Bishop et al., 2005; Chatanantavet and Parker, 2005; Wobus et al., 2006; Berlin and Anderson, 2007; Crosby et al., 2007; Gasparini et al., 2007). Although these approaches can produce an upstream-propagating wave in the landscape, they do not explicitly model the erosion processes that occur at a waterfall. More process-based models involve undercutting or undermining of the waterfall headwall from either plunge-pool erosion (Stein and Julien, 1993; Bennett, 1999; Alonso et al., 2002; Stein and LaTray, 2002; Flores-Cervantes et al., 2006) or seepage erosion (Howard and McLane, 1988; Dunne, 1990; Lobkovsky et al., 2007; Luo and Howard, 2008). The most well-known example is probably Niagara Falls, where a more resistant limestone cap rock appears to be progressively undercut by plunge-pool erosion, leading to collapse and upstream propagation of the waterfall (Gilbert, 1907).

Although undermining models can explain the persistence of some waterfalls, many waterfalls exist that are not undercut (von Engeln, 1940; Young, 1985). Many waterfalls also lack the prominent strong-over-weak stratigraphy that allows a vertical headwall to persist in weakly cohesive sediment (Holland and Pickup, 1976; Gardner, 1983). The undermining model has even been questioned for Niagara Falls, where progressive breakdown of the exposed bedrock face might be limiting headwall retreat instead of plunge-pool erosion (Philbrick, 1970, 1974). A rock-breakdown control on the rate of waterfall propagation has also been suggested for retreated waterfalls in Australia (Seidl et al., 1996; Weissel and Seidl, 1997) and Kaua‘i, Hawai‘i (Seidl et al., 1994, 1997). On Kohala, Hawai‘i, others have suggested that headwalls persist and retreat upstream in the absence of undercutting due to continual generation of stepped waterfalls and downward abrasion by gravel transport (Howard et al., 1994; Lamb et al., 2007). Although seepage erosion has been offered as an alternative to plunge-pool erosion (Kochel et al., 1985; Laity and Malin, 1985; Baker, 1990; Pederson, 2001), evidence in support of seepage erosion in bedrock is often ambiguous (Craddock and Howard, 2002; Lamb et al., 2006), and seepage flow is often incomplete to excavate collapsed debris away from the headwall (Lamb et al., 2006, 2007, 2008).

Whereas most landscape-evolution models treat bedrock as homogeneous (e.g., Howard et al., 1994; Whipple and Tucker, 2002; Berlin and Anderson, 2007), the mechanics of waterfall erosion appear to be strongly controlled by bedrock strength, jointing, and stratigraphy (Young, 1985; Miller, 1991; Weissel and Seidl, 1997; Stein and LaTray, 2002; Frankel et al., 2007). Riverbed erosion by abrasion or plucking in general depends on rock strength and joint orientations and spacings (Annandale, 1995; Hancock et al., 1998; Whipple et al., 2000; Coleman et al., 2003; Wohl, 2008). In bedrock with pervasive vertical joints, it has been suggested that waterfalls might retreat due to toppling of rock columns in the absence of undercutting (Young, 1985; Seidl et al., 1996; Weissel and Seidl, 1997). Such waterfalls may be associated with particular geologic settings where bedding or foliation planes have been tilted to near vertical (Weissel and Seidl, 1997; Frankel et al., 2007), or in volcanic terrains where flood basalts develop cooling joints (i.e., columnar basalt) (Young, 1985; O‘Connor, 1993; Lamb et al., 2008). For example, Weissel and Seidl (1997) showed clear evidence of toppled columns at the headwall of Apsley River Gorge, Australia, where subvertical joints in metasedimentary rocks promote rotational failure. Near-vertical fracturing also might be expected around bedrock canyons in noncompressive tectonic settings due to
Figure 1. (A) Shaded relief map of the central part of the Snake River Plain, Idaho, showing the locations of three canyons with steep headwalls: Malad Gorge, Box Canyon, and Blue Lakes Canyon. The 30-m-resolution topographic data are from the U.S. Geological Survey. The black lines are 100 m contours. The inset map shows the study location in central Idaho, USA. (B) Shaded relief map of Malad Gorge. C and D labels show the location of the canyon heads in the following photographs. The 10-m-resolution topographic data and 1-m-resolution orthophotograph are from the U.S. Geological Survey. The black lines are 10 m contours. (C) Photograph of one of the heads of Malad Gorge. The headwall relief is ~50 m. (D) Photograph of the head of Woody’s Cove. A person for scale is circled in the upper right. (E) Shaded relief map of Blue Lakes Canyon. The 10-m-resolution topographic data and 1-m-resolution orthophotograph are from the U.S. Geological Survey. The black lines are 10 m contours. (F) Shaded relief map of Box and Blind Canyons. G label shows the location of the canyon head in the following photograph. The 1-m-resolution airborne laser-swath mapping data were collected by the National Center for Airborne Laser Mapping, and the 1-m-resolution orthophotograph is from the U.S. Geological Survey. (G) Photograph of the headwall of Box Canyon. The headwall relief is ~35 m. All topographic data are projected onto UTM 11N with NAD83 datum.
In this paper, we explore the hypothesis that waterfalls can persist in the absence of undercutting where river flows generate drag forces sufficient to induce toppling in fractured rock and transport collapsed debris away from the headwall. First, we present a model that balances torques on a rock column. This model reveals that a vertical headwall is likely to be maintained during upstream waterfall propagation where columns are tilted in the downstream direction or slightly tilted in the upstream direction. The model also can be used to estimate the flow discharge needed to cause toppling and headwall retreat. Second, predictions are compared to experiments where stacked bricks failed in a laboratory flume. Finally, we discuss the implications of this work for canyons cut into columnar basalt on Earth and Mars. In particular, we focus on several canyons in the Snake River Plain of Idaho, United States, that have been cut by large paleofloods into columnar basalt (Fig. 1). These include Blue Lakes Canyon, a cataract of the Bonneville Flood (Malde, 1968; O’Connor, 1993), Box Canyon, cut by a paleoflood ca. 45 ka (Lamb et al., 2008), and Malad Gorge, cut by the Wood River (Kauffman et al., 2005) (Fig. 1).

**TORQUE-BALANCE MODEL**

We develop our model for a bedrock headwall with three perpendicular joint sets, where two are normal to the land surface and the other is parallel to the land surface (Figs. 2 and 3). For convenience, these joint sets will be referred to as vertical and horizontal joints herein. In columnar basalt, for example, these could be contraction cooling joints and bedding planes, respectively (e.g., Aydin and DeGraff, 1988; Budkewitsch and Robin, 1994). For simplicity, we assume that vertical joints extend unimpeded through horizontal joints and vice versa, essentially creating bedrock columns that are composed of individual blocks stacked on top of one another (Fig. 2). We also make the important simplifying assumption that the fracture-bound blocks are free standing and not interlocked with neighboring columns. We discuss the limitations of this approach in the Discussion section. Despite these simplifications, we hypothesize that the geometry used here should be applicable to many landscapes where jointing is pervasive because there should be a high probability that some joints align and are sufficiently wide that they outline free-standing rock columns. This seems to be reasonable based on our field observations of extensive vertical joints with large centimeter-scale joint diameters in flood basalts exposed within the Snake River Plain, Idaho (Fig. 1).

To begin, we first balance the forces and torques caused by flow across a single column of rock at a waterfall lip (Fig. 3). Gravity tends to hold a column in place, whereas it is destabilized by shear and drag from overspilling water and buoyancy produced by the plunge pool at the base of the headwall. Pressure due to heightened pore-fluid levels between rock columns is neglected here, but it is considered in the Discussion section. After balancing torques on a single rock column, we next consider the likely possibility that a column is composed of several blocks stacked vertically on top of one another (Fig. 2). We use the torque-balance model to investigate the conditions where an entire column is more likely to fail than any given block that comprises the column. Where this is the case, a steep headwall is likely to persist during upstream waterfall retreat.

Subject to sufficient force at the top of a rock column, it might fail by either sliding or toppling. Larger aspect ratios of block height ($H$) to width ($L$) favor toppling over sliding (Terzaghi, 1962; Goodman and Bray, 1977). For blocks of the same mass and volume, taller blocks have a larger torque arm (Fig. 3), increasing the likelihood of toppling, and a larger normal stress and frictional resistance at their base, decreasing the likelihood of sliding. For example, if roughness within a joint is parameterized using a friction angle of 45°, toppling is favored for $H/L > 0.5$ (Selby, 1993). In this paper, we limit our analysis to blocks or columns that have sufficiently large aspect ratios (or high basal friction) so that failure occurs by toppling, not sliding.

The forces per unit width due to gravity $F_g$, buoyancy $F_b$, shear $F_s$, and drag $F_d$ acting on a column of rock (Fig. 3) can be calculated as

$$F_g = \rho g L H,$$  \hspace{1cm} (1A)

$$F_b = \rho g L H,$$  \hspace{1cm} (1B)

$$F_s = \tau_s L,$$  \hspace{1cm} (1C)

$$F_d = \frac{1}{2} \rho C_d U^2 \eta,$$  \hspace{1cm} (1D)

where $\rho$ and $\rho$ are the densities of rock and water, respectively, $g$ is the acceleration due to gravity, $\tau_s$ is the boundary shear stress at the top of the column at the overfall lip, $C_d$ is a drag coefficient of order unity (Batchelor, 1967), $U$ is the velocity of the flow spatially averaged over the area of the column (per unit width) that protrudes into the flow a distance $\eta$, and $H_p$ is the depth of the plunge pool. In formulating the buoyancy force, we have neglected the orientation difference between the submerged portion of the rock column and the free surface of the plunge pool, which tends to be perpendicular to gravity. This was done because it greatly
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simplifies the torque balance that follows (e.g., because the center of buoyancy changes as a function of plunge-pool height) and because we found that accounting for this orientation difference had a relatively small effect on the model predictions.

The resulting torques (per unit width) that produce rotation about the pivot point \( P \) (Fig. 3) can be calculated from the product of these forces and a representative torque arm:

\[
T_s = \frac{1}{2} F_p L \cos \left( \frac{H}{L} \tan \theta \right),
\]

(2A)

\[
T_v = \frac{1}{2} F_v L,
\]

(2B)

\[
T_T = F_T H,
\]

(2C)

\[
T_d = F_d H,
\]

(2D)

To assess stability, we define a factor of safety (FS) as the ratio of resisting to driving torques as

\[
FS = \frac{T_r - T_v}{T_r + T_d}.
\]

(3)

Therefore, where \( FS \leq 1 \), a column is predicted to fail, and where \( FS > 1 \), a column is stable. Combining Equations 1, 2, and 3 results in

\[
FS = \frac{\rho g L}{\rho g L} \left( \frac{1 - H}{L \tan \theta} \right) - \frac{H_p}{H},
\]

(4)

which can be used to predict the stability of a single column of rock.

In many cases in nature, a column of rock is actually composed of multiple stacked blocks separated by horizontal joints or bedding planes (Fig. 2). In order for a vertical headwall to persist during upstream propagation, the entire column must fail before any given block contained within the column. For example, is toppling failure more likely to occur in Figure 2 about pivot point \( P_1 \), \( P_2 \), or \( P_3 \)? If toppling occurs about point \( P_1 \) or \( P_3 \), then one might expect the headwall to diminish in height over time or evolve into a stair-step pattern. Alternatively, if failure occurs about \( P_1 \), then the vertical headwall should persist during upstream propagation. To assess the relative stability of a stack of blocks, we let the height of a block \( H_1 \) above any arbitrary pivot point be \( H_1 = \gamma H \), where \( 0 \leq \gamma \leq 1 \). For example, in Figure 2, \( \gamma = 2/3 \) for rotation about \( P_1 \) and \( \gamma = 1/3 \) for \( P_2 \). The factor of safety for a block of height \( H_1 \) (FS) normalized by the factor of safety for the entire column (i.e., Eq. 4) can be written as

\[
FS_{\gamma} = \frac{\rho g L}{\rho g L} \left( \frac{1 - \gamma H}{L \tan \theta} \right) - \frac{H_1}{H} \left( \gamma \right),
\]

where \( H(\gamma - 1) + H_1 \geq 0 \) accounts for the fact that the elevation of the plunge pool is measured from the base of the column.

Inspection of Equation 5 reveals that the relative stability of any given block in the column is a function of the initial tilt angle of the column (\( \tan \theta \) and the plunge-pool height \( H \)). For example, if the buoyancy force is negligible (i.e., \( H_p = 0 \)) and a column is perfectly level (\( \tan \theta = 0 \)), then relative stability does not depend on block height, \( FS_{\gamma} / FS = 1 \), and failure is predicted to be equally likely at any pivot point. This is because both the force induced by the overflowing water (which destabilizes the column) and the weight of the column (which adds stability) depend linearly on block height. For conditions where the column is tilted upstream (i.e., \( \tan \theta > 0 \) and \( H_p = 0 \)), the uppermost block (i.e., small \( \gamma H \)) is predicted to be less stable than any taller block, including the entire column (i.e., \( FS_{\gamma} / FS < 1 \)). During upstream propagation, this result should be broken down of the vertical headwall. Alternatively, a downstream tilt angle (\( \tan \theta > 0 \)) favors toppling of the entire column over any given block (i.e., \( FS_{\gamma} / FS > 1 \)), which would allow the vertical headwall to persist during upstream propagation.

The relative stability of the column is also predicted to be a function of the plunge-pool height. Where \( 0 < H_p < H \), a greater portion of the entire column is submerged than any block higher up, which results in a larger destabilizing buoyancy force acting on the entire column. Where \( H_p > H \), the entire column is submerged, and therefore the buoyancy force affects all blocks equally.

Equation 5 implies that vertical headwalls can persist in the absence of undercutting in fractured rock where columns are tilted in the downstream direction, or where slightly tilted upstream if \( 0 < H_p < H \). This might often be the case for canyons cut into columnar basalt, where canyon-cutting floods would follow the same topographic gradient as the lava flows that preceded them, and near-vertical joints (e.g., due to cooling) tend to open perpendicular to the land surface (e.g., Budkewitsch and Robin, 1994). These predictions are explored in more detail next by comparing them to results from flume experiments.

**FLUME EXPERIMENTS**

**Experimental Setup and Methods**

Experiments were performed in a 5-m-long, 30-cm-wide flume at the Richmond Field Station of the University of California to test the predictions of the toppling model. The flume could tilt from 2 deg to 10 deg, and the discharge was adjustable using a variable frequency–controlled pump. A backward-facing, 19.2-cm-high, vertical step was installed ~2 m from the inlet of the flume, creating a waterfall in the flume (Fig. 4). The flume bed and waterfall were made of wood and were relatively smooth. To prevent development of a zone of low air pressure behind the waterfall (e.g., Chanson, 2002), a false wall narrowing the flume width to 27 cm was installed upstream, but not downstream, of the waterfall. Thus, as water poured over the step, it separated not only from the vertical wall below but also from the sidewall, allowing the air pocket beneath the falling water to remain at atmospheric pressure.

Clay bricks of three different thicknesses \( (L = 3 \text{ cm}, 6 \text{ cm}, \text{ and } 9.7 \text{ cm}) \) and a saturated density \( (\rho_p = 2240 \text{ kg/m}^3) \) were used to simulate bedrock columns. These bricks were placed just downstream of the step with their long dimension \( (19.5 \text{ cm}) \) parallel to the width of the flume and their short dimension (i.e., \( L \)) parallel to the length of the flume (Fig. 4). The bricks were 9.7 cm in height, and two bricks were stacked on top of one another to create a total column height of \( H = 19.4 \text{ cm} \). Only one column of bricks (i.e., two bricks in total) was analyzed per experiment. The orientations were intentionally chosen for large \( H/L \) to encourage toppling over sliding. Within the flume, the bricks were placed on a piece of sheet metal that could be tilted to change the angle of the brick relative to horizontal (\( \theta \)) and raised to adjust the distance the bricks protruded above the step (\( \eta \)) (Fig. 3).

For a set flume-bed slope \( (S = \tan \beta \text{ where } \beta \text{ is the angle between the floor of the flume and horizontal}) \), brick angle (\( \theta \)) and protrusion height (\( \eta \)), the discharge of flow was increased stepwise by ~0.7 L/s, pausing for ~90 s between adjustments, until the bricks toppled (Fig. 3B). This time scale was chosen to be significantly longer than the time scale of turbulent eddies (e.g., \( h(\tau_p)^{1/3} \), where \( h \) is the flow depth). The results in Table 1 are classified based on whether the top brick failed or the entire column toppled, as in Figure 4B. The plunge-pool depth \( (H_p) \) and flow depth \( (h) \) were measured using a ruler.

Each experiment was designed to include a significant component of drag by setting \( \eta > 0 \) because the shear force alone was not sufficient to induce toppling within the discharge constraints of the flume. For most experiments \( \eta/h \) was small (Table 1), and the protruding column did not affect the overflowing water or the development of the plunge pool (e.g., Fig. 4). This was not true, however, for the experiments with the widest bricks \( (L = 9.7 \text{ cm}) \) where \( \eta \) needed to be
large to induce toppling. For these experiments, the overflowing water did not appear as in other experiments, but instead was deflected upward and projected downstream upon impact with the protruding column, resulting in a smaller than expected plunge-pool depth (Table 1). The approximation of the water surface is drawn as a blue line. (B) Two blocks after failure. The blocks failed together as a column, and the top block slid to the right upon impact. Note the sheet metal that the bricks were placed on to adjust θ and η.

Figure 4. Photographs from experiments designed to test the toppling model. (A) Two blocks (L = 6.0 cm) stacked on top of one another in front of the backward-facing step before failure occurred. The step height (H = 19.2 cm) and plunge-pool height (H_p) are noted. The approximate location of the water surface is drawn as a blue line. (B) Two blocks after failure. The blocks failed together as a column, and the top block slid to the right upon impact. Note the sheet metal that the bricks were placed on to adjust θ and η.

It is difficult to estimate the shear stress at a waterfall since the flow is accelerating over the lip. Instead, we calculated the shear stress from the waterfall since the flow is accelerating over the waterfall (τω) to the shear stress upstream (e.g., following Stein and Julien, 1993; Flores-Cervantes et al., 2006; Haviv et al., 2006) as

\[ \frac{\tau}{\tau_o} = \frac{\rho C_{f1} U_o^2}{\rho C_{f2} U^2} = \alpha^2, \]

where \( C_{f1} \) is a friction coefficient. Equation 7 implicitly assumes that drag due to the protrusion of bedrock columns into the flow does not affect the partitioning of stress upstream of the waterfall. By letting the flow velocity about the protruding portion of the column \( U_p \) follow a similar scaling, we can write

\[ \frac{U_h}{U} = \frac{\sqrt{\tau_o / \rho C_{f1}}}{\sqrt{\tau_o / \rho C_{f2}}} = \alpha \frac{C_{f2}}{C_{f1}}, \]

where \( C_{f1} \leq C_{f2} \) because \( U_o \) is averaged over a lower portion of the water column than \( U \).

Experimental Results

The conditions needed to induce toppling in our flume experiments are shown in Table 1. In the experiments, flow depth upstream of the waterfall varied from 3 \( \leq h \leq 13 \) cm and velocity varied from 0.8 \( \leq U \leq 2.7 \) m/s. The flume-bed slope ranged from 0.0083 \( \leq S \leq 0.097 \), and all runs had supercritical flow upstream of the waterfall with 1.14 \( \leq Fr \leq 3.48 \). The angle of the brick column was varied independently of the flume-bed slope (Table 1). The plunge-pool depth ranged from \( H_p = 0 \) cm to fully submerged at \( H_p = 19 \) cm. In order to apply the model, the torque due to drag at the waterfall lip was calculated from Equations 2 and 8, where we assumed \( C_{f2} = C_{f1} \) and \( C_{f1} = 1 \) for simplicity.

Figure 5 shows the results of the measured torque balance at failure for each experiment without differentiation as to whether one or two bricks failed. The results are clustered with respect to gravitational torque, where the wider and taller columns at the time of failure required more torque to destabilize. Drag dominated over shear in all cases, which was intentionally designed into the experiments by letting \( η > 0 \). The data are more scattered for the widest bricks (\( L = 9.7 \) cm) because \( η/h \) was large and the flow and plunge pool were significantly affected by the column protrusion (as discussed in the Experimental Setup section). The model predictions (i.e., Equation 4 with \( FS = 1 \)) match
the experimental results well. In most cases, the sum of the torque due to shear, drag, and buoyancy at the point of toppling equaled the expected stabilizing torque due to gravity.

Figure 6 shows the experimental results and predictions for the relative stability of the top brick versus the entire column (i.e., Eq. 5 with $\gamma = 0.5$) for the case $L = 3$ cm. The results show two distinct regimes favoring either toppling of the entire column or the top brick, with the transition occurring at about $\tan \theta = -0.03$. As illustrated in the Torque-Balance Model section, in the absence of a differential buoyancy force (i.e., $F_B = 0$ or $H_a = H$), the transition would be expected at $\tan \theta = 0$. The finite plunge-pool depth, however, favors toppling of the entire column, which shifts the transition to negative brick angles. These experimental results match the predictions well, both in the magnitude of $F_S/F_S$ and in the relative stability of the column versus the top brick. Because the relative stability depends on the plunge-pool depth ($H_p$), the predicted values of $F_S/F_S$ were calculated using the average measured plunge-pool depth for each set of experiments.

### FIELD APPLICATION

Equations 4 and 5 can be used to predict the geometry of upstream-propagating headwalls and the magnitude of flow necessary to induce toppling in natural systems. It is useful to explore the parameter space with respect to column aspect ratio ($H/L$) and tilt angle ($\theta$) to describe the expected geometry of propagating waterfalls in fractured rock (Fig. 7). The aspect ratio $H/L = 0.5$ separates the fields of toppling from sliding assuming a friction angle of 45° along horizontal joint surfaces (Selby, 1993). The sliding regime will necessarily produce a more diffusive or stair-step headwall. Within the toppling regime, a block will fail due to gravity alone (in the absence of water flow) if it is tilted beyond a critical angle given by $\tan \theta > L/H$ (i.e., where $T = 0$ in Eq. 2) (Fig. 7) (Goodman and Bray, 1977). For gravitationally stable blocks, positive tilt angles (tan $\theta > 0$) always favor toppling of the entire column and therefore should preserve a steep headwall during upstream propagation. Steep headwalls can also persist at negative tilt angles, but this depends on the spacing of horizontal joints within a column (i.e., $\gamma$) and the relative height of the plunge pool (i.e., $H/H$), as described below.

By setting $F_S/F_S = 1$, Equation 5 can be simplified to find the boundary between the vertical and stair-step headwall regimes as

$$\sin \theta = -\frac{H_a}{H} L \left(\frac{1}{\gamma}\right)$$

Note: The field "toppled" denotes the number of bricks that toppled during an experiment, where "2" indicates that both bricks toppled as a column. See text and notation list for definitions.
The largest difference in buoyancy between the entire column and any individual block occurs if the plunge-pool elevation is at the same level as the joint that defines the bottom of the block of interest (i.e., $H_p = H(1 - \gamma)$), so that the entire top block is not submerged. Inserting this condition into Equation 9 results in the maximum (negative) angle possible for toppling of the entire column:

$$\sin \theta = \frac{\rho L}{\rho H}.$$ (10)

Therefore, for $\sin \theta < -\frac{\rho L}{\rho H}$, a headwall is predicted to form a stair-step pattern (Fig. 7).

For $-\frac{\rho L}{\rho H} < \sin \theta < 0$, either a stair-step pattern or a vertical headwall can result, and this depends on Equation 9.

Where toppling is likely to occur, Equation 4 can be used to predict the magnitude of flow necessary to induce failure. To illustrate this, we examine the deeply incised canyons with near-vertical, amphitheater headwalls that exist as tributary canyons to the Snake River Canyon in central Idaho (e.g., Malad Gorge, Blue Springs Canyon, and Box Canyon) (Fig. 1A). All of these canyons are thought to have formed by catastrophic paleofloods of various sources during the Quaternary that poured over the wall of the Snake River Canyon creating upstream-propagating waterfalls. Malad Gorge has three main canyon heads (e.g., Figs. 1B, 1C, and 1D) that were probably carved by large flood events of the Big and Little Wood Rivers (Kauffman et al., 2005). Blue Springs Canyon (Fig. 1E) was cut by the Eden Channel of the great Bonneville Flood, which catastrophically drained paleo-Lake Bonneville at ca. 15 ka (Malde, 1968; O’Connor, 1993). Box Canyon (Figs. 1F and 1G) was cut ca. 45 ka by a megaflood that probably initiated in the Wood or Lost River drainage basins (Lamb et al., 2008).

The Quaternary basalt that makes up the canyon walls ranges in age from ca. 80 ka to 400 ka (Tauxe et al., 2004; Kauffman et al., 2005; Aciego et al., 2007), is blocky and hard, and shows no visual evidence of significant chemical weathering or undercutting (Figs. 1C, 1D, and 1G). The headwalls of these canyons are on the order of 30–50 m high, and the basalt contains near-horizontal joints along bedding surfaces and near-vertical cooling joints, with spacings between joints of ~0.5 m (Figs. 1C, 1D, and 1G). Measurements of boulder sizes on the floor of Box Canyon, for example, yielded a median boulder size of 0.3 m (Lamb et al., 2008), which is consistent with our visual estimate of joint spacing. The landscape upstream
of these canyon heads is relatively flat, but it does dip toward the canyons with a slope of ~1\% (Fig. 1A). Because bedding planes are probably on average parallel to the land surface, and near-vertical joints are perpendicular to the land surface (Budkewitsch and Robin, 1994), Equation 5 predicts, given sufficient flow, that rotational failure should occur at the base of the columns, preserving their steep headwalls (i.e., tan \( \theta = S > 0 \); Fig. 7). Therefore, Equation 4 can be used to estimate the flow conditions needed to cause toppling failure of rock columns that extend the total relief of the canyon headwalls.

First, it is useful to rewrite the flow velocity in Equation 4 in terms of total bed stress (using Eq. 8) and to assume steady and uniform flow (i.e., \( \tau_T = \rho ghS \)) upstream from the headwall. This modification results in

\[
FS = \frac{\rho L \cos \theta}{2 \alpha \eta} \left(1 - \frac{HS}{L} \right) - \frac{H_p}{H} ,
\]

where \( \alpha \) can be simplified to

\[
\alpha = 0.4 \frac{C_{fl}}{S} + 1 \quad \text{for } Fr > 1 \quad \text{and}
\]

\[
\alpha = 1.4 \left( \frac{C_{fl}}{S} \right)^{1/3} \quad \text{for } Fr < 1 .
\]

All variables in Equations 11 and 12 can be estimated or measured from the topography and geometry of the canyons except the plunge-pool depth. The depth of water behind a waterfall at a vertical drop can be calculated following theory for a two-dimensional (2-D) plane jet impinging on a horizontal plane at steady state (Leske, 1963; Vischer and Hager, 1995; Chanson, 2002):

\[
\frac{H_p}{H} = \frac{1 + 2(1 - \cos \delta)Fr^2}{2 + Fr^2 + 2\alpha^2 S \eta / h} ,
\]

where \( \cos \delta = \left[ 2 + Fr^2 - 2(1 - \cos \delta)Fr^2 \right]^{1/2} \)

is the angle of the jet relative to horizontal, and \( Fr^2 = \alpha^2 S \eta / C_{fl} \) is the Froude number at the overflow point squared. Equation 13 provides a good prediction of the observed pool heights in our flume experiments (Fig. 8), except for the cases with large \( \eta / h \) (i.e., \( L = 9.7 \) cm), where plunge-pool development was disturbed.

Equations 11–13 reveal that the flood stage necessary to produce toppling failure (i.e., \( FS = 1 \)) normalized by the column height is a function of six dimensionless parameters: \( H/L, S, \eta/L, \rho / \rho, C_{fl}, \) and \( C_f/C_{fl} \). For the basalt columns of interest, we set \( \rho / \rho = 2.8, C_{fl} = C_f \).
depths upstream of the waterfall are 1.6 < h < 3 to 9 m, and the calculated flow plunge-pool depths at the threshold of toppling that the flood flows at Box Canyon would have entered motion considerations for boulders within the canyon (Lamb et al., 2008). Thus, it seems that the flow upstream of the canyon head and incipient motion considerations for boulders within the canyon (Lamb et al., 2008). Thus, it seems that the flow at Box Canyon would have been competent to induce toppling and preserve a steep headwall, as well as transport collapsed boulders out of the canyon (Lamb et al., 2008). This necessary flood stage (Fig. 9) also seems reasonable for Malad Gorge and Blue Lakes Canyon given the magnitude of paleoflood events that have occurred there (Kauffman et al., 2005; O’Connor, 1993).

DISCUSSION

Interlocking

Our model and flume experiments of headwall retreat by toppling are for simple joint patterns that probably do not translate to all cases in nature. For example, within a column of rock at a headwall, some blocks might extend between neighboring columns, and this may induce a yield strength that must be overcome for failure to occur. This could impact the predictions of flow needed to induce toppling and the morphology of the headwall (i.e., Eqs. 4 and 5). If yield strength varies with column height, then it is possible that a given block might fail before the entire column, even with a positive tilt angle. This appears to be the case for Apsley Gorge, Australia, where subvertical joints in metasedimentary rocks control the morphology of the headwall there (Weissel and Seidl, 1997). Although dominant joints dip in the downstream direction, the connectivity of joints apparently is not sufficient to create a vertical or overhanging headwall. Instead, toppling of blocks and overhangs occur at a local scale, whereas overall, the headwall is convex in profile (Weissel and Seidl, 1997). On the other hand, jointing appears to be pervasive enough in the basalt walls of the Idaho canyons that some vertical joints connect between stacked lava flows and outline free-standing columns of rock (Figs. 1C, 1D, and 1G).

Quantitative assessments of the strength of interlocking along vertical joints are difficult (Whipple et al., 2000). Unlike horizontal joint surfaces, where resistance to sliding can be calculated from the product of a friction angle and the weight of the overlying rock column (Terzaghi, 1962; Hancock et al., 1998), there is not a characteristic normal stress acting on vertical joints. Instead, the yield strength (if it exists) must be due to a sum of interlocked roughness elements within a joint plane, each of which has highly variable directions and magnitudes of local stress. The yield strength of interlocked columns probably depends on the roughness of joint surfaces, bedrock type, rock strength, degree of weathering, and fracture geometries. There is a clear need for future work to measure the yield strength of bedrock joints at field scale.

Pore Pressure

In addition to the forces already discussed, pore pressure might also be important for causing failure of a rock column. Pore pressure might be induced at a waterfall headwall due to infiltration and groundwater flow toward the headwall. A differential pore-water height across a column of rock would produce a force and a torque on the column, and if this is significant, it could induce toppling. This necessarily makes our factor of safety analysis conservative because it does not include pore-pressure gradients. Like buoyancy, torque caused by pore fluids should also favor toppling of the entire column over any individual block higher up in the column because hydrostatic water pressure scales linearly with water depth.

To first order, pore-pressure gradients can be estimated by assuming one-dimensional, steady-state Darcy flow within an unconfined aquifer (i.e., the Dupuit approximation; Freeze and Cherry, 1979).
were \( \approx 1 \times 10^{-2} \) m/s, peak rates in the first several hours of each test. When we let \( h_i \) be the elevation of pore water within a vertical gap immediately before the waterfall and neglect the plunge-pool height, then

\[
\frac{dh_i}{dx} = -\frac{h_i}{L},
\]

and Equation 14 can be rewritten as

\[
h_i = \sqrt{\frac{V}{K}},
\]

where \( V_i = q_i/D_i \) is the steady-state infiltration rate, and \( D_i \) is the surface area per unit width over which infiltration occurs. Thus, pore pressure should be important for large infiltration rates, large infiltration distances, and small horizontal hydraulic conductivity.

Our small-scale experiments were intentionally designed to limit pore-pressure effects by making \( D_i \) small (i.e., only one vertical crack). This was done because \( V_i/K \) was large in our experiments due to the relatively large ratio of vertical joint spacing to column width, and \( V_i \) was small because we lacked joint planes parallel to the downstream direction of the flume. These effects were due to the small scale of our experiments and are probably not representative of the canyons of the Snake River Plain. For example, horizontal conductivity is large in the Snake River Plain owing to rubble zones, fissures, fractures, and lava tubes (Knutson et al., 1992), and average values of \( K \) are \( \approx 5 \times 10^{-3} \) m/s, with maximum values as high as \( K = 1 \times 10^{-2} \) m/s (Ackerman, 1991; Malde, 1991; Wellman and Reed, 1997). The headwall of Box Canyon, for example, is the site of one of the largest springs in the United States, with a discharge of \( \approx 9.5 \) \( \times 10^{-3} \) m/s (Fig. 1B). Pond tests in the eastern Snake River Plain indicate average infiltration rates over several days of \( \approx 5 \times 10^{-2} \) m/s; peak rates in the first several hours of each test were \( \approx 1 \times 10^{-2} \) m/s (Faybishenko et al., 2000; Unger et al., 2004). Thus, for the Snake River Plain, \( V_i/K \approx 10^{-1} \). Using this value and setting the column width to \( L = 1 \) m in Equation 15, the differential pore fluid height is calculated to exceed \( h_i \approx 1 \) m, even if infiltration took place over a distance of \( D_i \approx 10 \) km upstream of the headwall. This suggests that differential pressure caused by pore fluid is probably not important in inducing toppling for the canyons of the Snake River Plain since the pore-fluid heights are small compared to both the column heights (\( H \)) and plunge-pool depths (\( H_p \)). This, however, might not be the case in other locations, especially where \( V_i/K \) is large.

**Lateral Focusing of Flow**

One of the limitations of our model is that both Equations 4 and 5 were formulated in terms of a unit channel width and neglect lateral changes in the flow. Spatial acceleration in flow might be important for the canyons of the Snake River Plain where unconfined flood water poured over the wall of the Snake River Canyon. In this case, the low pressure associated with overflow at the canyon head might have caused flow to focus toward the canyon head, resulting in faster flow there, and an increase in the rate of toppling and canyon-head retreat (e.g., by increasing \( V_i \) and \( H_p \) in Equation 4). This also makes our estimates of flow discharge needed to cause toppling conservative. Feedback between flow focusing and canyon-head retreat might explain why these flood events resulted in the formation of tributary canyons rather than broad escarpments. Furthermore, this mechanism might also explain the amphitheater planform of these canyon headwalls, such that toppling was more likely at the center of the headwall due to flow focusing, whereas the edges of the headwall were at or near the threshold of toppling. These three-dimensional effects need to be explored in future experimental and numerical model studies.

**Implications for Mars**

The surface of Mars has abundant canyons with steep amphitheater-shaped headwalls and knickpoints similar to those described here (e.g., Baker, 1982). Although these features are often attributed to seepage erosion (e.g., Baker, 1990; Malin and Carr, 1999; Harrison and Grimm, 2005; Luo and Howard, 2008), recent work suggests that waterfall erosion might be a more plausible mechanism, especially in hard rock (Crandock and Howard, 2002; Howard et al., 2005; Lamb et al., 2006, 2007, 2008). This is supported by the observation that many Martian valleys have shallow tributaries upslope, suggesting that overspill occurred in the past (Irwin et al., 2004; Crown et al., 2005). Retreating waterfalls might have been initiated by flood water breaching the rim of an impact crater (e.g., Irwin and Howard, 2002; Howard et al., 2005) or the rim of a pre-existing river canyon (as in Idaho; Fig. 1). For example, the 900-km-long Ma’adim Vallis was probably carved by catastrophic overflow of a highland lake resulting in a 300-m-high knickpoint in the main channel and hanging tributary canyons (Irwin et al., 2002, 2004). As another example, Williams and Malin (2004) described a 100-m-high near-vertical headwall of a channel within Kasei Valles that appears to represent a relict retreating waterfall. We hypothesize that the abundance of amphitheater-headed canyons and steep knickpoints on Mars might reflect a well-jointed lithology (e.g., columnar basalt), which is susceptible to toppling failure (and flood events sufficient to induce failure and transport collapsed material) rather than seepage erosion. Images of the surface of Mars have revealed layered material in canyon walls, and spectral data suggest that some of this is olivine-rich basalt (Bandfield et al., 2000; Christensen et al., 2003; Hamilton and Christensen, 2005). If our hypothesis is correct and fracture patterns can be observed or estimated, Equation 4 and Figure 9 can be used to calculate the flow required to cause toppling failure at the headwalls of Martian canyons. Because Equation 4 is based on a ratio of torques, the effects of lower Martian gravity are implicitly accounted for in the model.

**CONCLUSIONS**

We hypothesize that waterfalls can persist during upstream headwall retreat in the absence of undercutting due to toppling in well-jointed bedrock. A torque-balance model indicates that a column of rock at a waterfall is generally less stable than any given block that comprises it because of buoyancy produced by the plunge pool at the foot of the waterfall, a downstream tilt of the rock column, or both. The model provides good predictions of the flow needed to induce toppling and the resulting morphology of the headwalls in flume experiments and the amphitheater-headed canyons of the Snake River Plain, Idaho, that were carved by large-scale floods. This suggests that erosion of bedrock canyons by waterfall retreat can be rapid where bedrock is well jointed and flow is sufficient to topple rock columns and excavate the collapsed material. Where block toppling can explain the origin of steep waterfalls and amphitheater-headed canyons on Earth and Mars, the torque-balance model can be used to constrain the minimum discharge of water needed to carve these features. Overall, our work adds to the growing recognition that bedrock fracture geometry can play a fundamental role in knickpoint morphology and retreat rate, and therefore it needs to be incorporated into landscape evolution models.

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